LETTERS TO THE EDITOR.

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The Teaching of Mathematics.

MANY correspondents who hold views much like my own are angry that I should lend my countenance to the sort of reform in mathematical teaching which is now being so strongly advocated by the recognised teachers and by mathematicians. I beg to assure these friends that I am acting in a very consistent way, and I mean to help the reformers so long as they do me the honour to let me assist them.

My dearest wish is that Englishmen should prepare for the new phase in the struggle for existence which has so suddenly come upon the world. I believe that it has been growing for sixty years, that it is going to be very intense during the next twenty years, that it will be important for the next hundred years and that the weaker nations will have been completely defeated before the end of that time, before people in general recognise their foolishness in wasting coal. At Glasgow I was sneered at as thinking that all men were going to be engineers. Would to heaven that I could think of all young Englishmen as being fit to become engineers! I firmly believe that no nation will survive the coming struggle which does not equip itself with that thorough training in applied science which I call engineering. I do not expect to be believed by our schoolmen. They do not know, and they would not care much if they did know. The study of natural science has been discouraged by clerical schoolmasters and ecclesiastics, and men like Huxley worried themselves very vainly in consequence. But the time is quickly coming when, instead of the castigating rod of Huxley, we shall feel the stinging scorpions which fate provides for all men who set themselves to believe lies. There is just one chance for us; it has befriended the English people at several critical times, namely, that however foolish we may be, other nations may be as foolish or even more foolish still; and possibly our people may attend, in good time, to the sort of advice which they so usually laugh at.

I say that if even a small amount of knowledge of natural science and of scientific method could be given to our public men it would be a good thing. Now all influential Englishmen pass their youth and get what is fondly called education at the public schools and the two older universities. I have satisfied myself that the authorities of these institutions will steadfastly set their faces against any such large and radical reform as I have asked for; I was quite sure of this before I gave my address at Glasgow. It was only during the discussion that I began to get hope of any reform whatsoever in these institutions, and now I am sure that, not only is a small immediate reform really possible, but that this reform will increase as the years go on.

It is true that I was thinking of other universities, of other schools and colleges, but if any reform, however small, is possible in these great institutions, surely it is the duty of every lover of his country to help forward this movement. When one thinks that Clifford and Cayley and Sylvester and the other great mathematicians of the British Association Committee of 1873 advocated for English public schools the very reforms in geometrical teaching which are now being asked for, and that in most of the public schools the teaching is even more stupefying now than it was in 1873, one is not likely to be very sanguine. But I have satisfied myself that there is a power now behind the reformers which is very much more earnest and persistent than Clifford could rely upon; indeed, I am satisfied that the reformers have with them the good wishes of every thoughtful teacher of the whole country. And after all, would not even the smallest of

reforms be exceedingly important in the public schools of England?

Eton and Harrow and Winchester and the rest, are they not great in every way? Oxford and Cambridge, what greater names are there than these among universities? These institutions in the past have educated the great men of England. I think I see immense faults in them, but when I compare them with all other schools of which I know anything, do I not know that in mental and bodily health their pupils are great in number and in quality? And if just now a time of strain is coming for which these institutions are quite unprepared, shall I sulk and say that because they will not do as I think they ought to do, then it is better that they should do nothing? To tell the truth, I think that our schools with a small reform will be equal to foreign schools even if these have every advantage that the latest notions in pedagogy can give them. Nevertheless, is it not rather the good material that comes to these schools for education than the schoolmasters of the schools that has our admiration? I feel myself that we are arming our people with bows and arrows when we might easily give them magazine rifles with telescopic sights. But surely, if it must be bows and arrows, and nothing better is possible, one is doing good by advocating the employment of the very best kind of bows and arrows.

I asked for a countenance from the mathematicians at Glasgow for a much more radical reform, not in the public schools and the two older universities—there I thought all reform impossible—but in the new universities, in all colleges where the study of natural science, and especially of applied science, is important. Although all the speakers were in favour of reform, quite half of them were out of sympathy with what I consider to be most essential. I know that they represent the great majority of the mathematicians and teachers of the country, and I should like again, however hopelessly, to explain my position to them. They refuse to attempt to look at things from my point of view. Their minds are beautifully in tune; what one scholar says, the mind of another responds to, but if anyone not a scholar of the orthodox type says anything, it is not heard at all, or only that part of the message is heard which is in tune with the receiving brain. My message must continue to seem to them very absurd if they make no effort to synchronise their mental apparatus with mine.

I assert that the orthodox logical sequence in mathematics is not the only possible one. I can imagine a sequence leading men of twenty-five to a proof of the axioms of Euclid; where it would start I do not know, perhaps in Berkeley's notions of sensation and that all matter and motion and shape are merely forms of consciousness. Surely every academic person will admit this as feasible. But he will not admit that there might be a thoroughly logical sequence starting with axioms which are now "proved" after many years of study, the notions underlying the infinitesimal calculus, for example; the notion that a map may be drawn to scale; the notions underlying the many uses of squared paper; the notion that decimals may be dealt with like ordinary numbers.

The swineherd Ho-ti and his son Bo-bo discovered the wonders of roast pork through the burning of their cottage, and they and their neighbours and every mandarin of China who studied the matter considered that it was absolutely necessary to burn a house down if one wanted roast pork. So the cult of house-burning arose in the land. But after many generations there came a sage of the name Pel-li, who pointed out to everybody that it was not necessary to burn houses, as a simple domestic fire was all that was needed. And he and his growing number of disciples were envied till a charge of impiety was brought against him. And of all the hundred accusers of Pel-li, all mandarins of high rank, every one was absolutely honest

and sure that it was really impious to roast pork without burning houses, and so Pel-li and his followers were crucified. If I were to tell my friends that I am Pel-li and that there is a very perfect parallelism between the two cases, they would laugh at the absurdity of such a statement, even if I made it from the cross. And yet, O mandarins, I say to you that you have brought against me the charge of impiety because you cannot imagine any other way of getting to the notions of the infinitesimal calculus than the way in which you yourselves arrived at them, and because I say that they are easier to take in than the axioms of Euclid.

Is it impossible, then, to imagine a different logical sequence from that in which one has been brought up? The very greatest difficulty which I meet with is in getting men to see that if a boy has practised measurement with a scale of inches and tenths of an inch he can understand decimals without a philosophical explanation. This fact, obvious to me, I have not been able to get believed in by any one teacher of arithmetic who plumes himself on his knowledge of the theory of teaching. Well, I go further and say that as an explanation is not necessary for a boy, to give him a grown-up explanation is a crime. Again, I like when dealing with quite easy arithmetic to make boys evaluate most complex formulæ, using all sorts of tabulated functions so that they may regard algebraic signs as a sort of shorthand. I cannot in a few words explain the wonderful mental value of this practice. My critics not only see no value in it, they look upon it with abhorrence. This is not through an effort of their reasoning; it is merely because the thing is strange to them, and, like the Dudley miner, they say, "''Ere's a stranger, Bill, let's 'eave a brick at him." If my critic has any kind of satisfying reason for his abhorrence I suppose it is because he thinks this new custom of mine resembles some most pernicious slipshod habits for which he is continually blaming his pupils—getting off propositions by heart and pretending one knows them ; using a rule of arithmetic or dynamics in a mechanical way without understanding why ; assuming that one understands a part of an investigation when one does not, and in all sorts of other fraudulent ways pretending to follow a logical sequence and degrading it. Now I also abhor these things. But what my students do is very different and is perfectly logical. They make no pretence of having proved anything, they are merely familiarising themselves with the shorthand of algebra, a thing that they cannot do too soon. If they get to look upon this as the A B C of mathematics they will not after many years of study feel proud of their mathematical knowledge when all they can do is to merely use formulæ in a text-book. Not long ago in an engineering journal the writer of a letter complained that he had been asked to evaluate the expression

$$a\epsilon^{-at} \sin(bt + g)$$

(being given the values of a, a, b and g) for several values of t. He said he had passed most difficult examinations and knew higher mathematics, but it was quite ridiculous that anyone should expect him to know so much. His anger was extreme. Now some of my students have evaluated things like this before they did any formal algebra at all, but they do not dream of calling it higher mathematics. Surely there is every good in letting a boy become familiar with all sorts of formulæ long before we lead him through the logical sequence which deals with such formulæ, just as we let a boy learn to use words before we teach him grammar or philology. But whether I am right or wrong, I do wish that my critics would try to see exactly what I advocate before they throw blame. The cock-shies that they fling their stones at have nothing in common with any part of my scheme.

I want it to be understood that I advocate a sequence as logical as the orthodox one, or rather, I should say, ever so much more logical, because in the orthodox sequence a boy is really unfamiliar with the ideas to which his so-called logic is applied. The usual sequence may be logical to a philosopher, but it is quite illogical to the average English boy.

I say that what is essential is that the student should be thoroughly familiar through experiment, illustration, measurement and every other possible method with the ideas to which he applies his logic. Also that the study should be of interest to him. I submit that the sequence which I recommend can really be made interesting to the average English boy, whereas the orthodox sequence is painfully uninteresting to him. One reason for this great interest lies in its immediate application to all sorts of actual problems such as he meets with in the study of natural science, and I do not care to hide the fact that there is a special interest which is due to the usefulness of the results of the study in the life-work that lie before him. If anybody cares he may misrepresent me here to any extent. Over and over again the academic person has been kind enough to sneer at my utilitarianism as if I were sacrificing the spiritual for the material, as if engineering were a thing of mere formula. On this I can add nothing to what I have already given in my British Association address and in my other papers. But if my critics only knew what wonderful regions of logical thought and high emotion are connected with the practical applications of natural science, if they had the respect which I confess to have for common things, they also might say as Heracleitus said of his kitchen with its pots and pans, " Here also are the gods." In his typical poem "Shop," Browning takes the college-common-room point of view. Heavens, what a sordid narrow point of view it is ! The Bloomsbury bric-à-brac shopkeeper gets all the poet's scorn because he does not hate shopkeeping, because to him "shop was shop only." Does anybody imagine that Shakespeare could not have glorified the life of the shop-man ? But this poet, with all the arrogance of his caste, says, "I want to know a butcher paints, a baker rhymes for his pursuit." The moment a man has leisure he must escape from his trade ! For my part, I believe that whatever a man finds to do he ought to do with all his might and with all love and devotion, or not attempt to do it at all. If he hates shop-keeping, let him give it up to someone else to whom shop-keeping is a perfect happiness. If Ruskin's influence over a man has been great enough to prevent his seeing the romance, the wonder of engineering, so that it is to him a mere trade by which he earns his bread and butter, in heaven's name let him give it up altogether and take to art criticism. For my lovely mistress, Applied Science, scorns a divided worship. It is disgusting to see young engineers who cannot compute, who know nothing of science, whose souls are not engrossed all the time with the greatness of their profession, who never think of their business after office hours, who think it all a mere matter of formulæ and tools. If they were fit for their work their lives would fill with happiness, and even the power to rhyme and paint and to create music might belong to them ; but woe unto the nation whose shopkeepers scorn shopkeeping while they paint or fiddle ; whose schoolmasters rely on cricket and a housemaster to do their proper business whilst they discuss Browning and the musical glasses. To make a man fit for, so that he may also love, his profession, is this a function to be scorned by schools and colleges? and am I to be sneered at as a utilitarian because I consider this a most important function of the schoolmaster?

Nobody has contradicted my statement that the orthodox method of cramming average boys with demonstrative geometry stupefies them and makes them hate mathematics all their life after. May I also point out that the beautiful philosophy of Euclid is also degraded, just as the literature of Greece and Rome is degraded, by our school methods. Is there anybody

who does not think of Euclid merely as a lower school subject ? At the British Association discussion a great mathematician was astonished that I should ever have had to study the fifth book of Euclid. He said he was more fortunate, because he was never taught it. Well, I was never compelled to study it, but I took to it through mere affection such as my critic deems it his good fortune never to have experienced. What I regret is that any kind of demonstrative geometry was given me when a boy, but since it was given me I am glad to think that I had Euclid's philosophy undefiled. I even dipped into those books now never published—the seventh, eighth, ninth, tenth, and also the thirteenth, fourteenth, and the books added by some Greek author whose name I forget, the fifteenth and sixteenth. At the same time, I feel that if demonstrative geometry is to remain a school subject for the average boy, it is absolutely necessary to replace the second and fifth books by algebra. The view to which I hold most firmly of all my views about the teaching of mathematics is that demonstrative geometry ought never to be taught to boys at all; it ought never to be taught in schools. It is a higher university subject. Euclid's treatment of proportion and of incommensurables is one of the most beautiful parts of that exact philosophy which the conventional schoolmasters are constantly seeking to degrade. The old philosophers thought that only a very few men of the most acute race that ever lived on this earth were fit to begin the study of geometry, and we use it as "an instrument for the cultivation of the mind" of the average young barbarian. Even my sense of the parlous state of the country cannot prevent me from grinning at the Rabelaisian humour of the position. Boys are not swine, but if you will force pearls upon them for food (poor boys, they do not know that the pearls are only cheap imitations) you must expect but small results either physically or spiritually. It must always be a pleasant memory to them, however, that they once did have pearls to trample under foot or to give them indigestion, and one may say that they are fairly safe from pearl hunger all the rest of their lives. Will any of my opponents deny that they ceased to study Euclid when they left school, except in the way of their trade as teachers? How many of them know anything of—I need not say Euclid's real philosophy—but even of modern geometry and the beautiful system of transversals developed by the Irish geometers? I recollect a lovely year of my life in which I was introduced to three new things—Tennyson's "Idyls" and McDowell's "Geometry" and Homer's "Odyssey" (Bohn's translation), and I hardly know even now which of the three gave me most pleasure. But I had had the good fortune not to have pearls forced upon me as a boy. Yes, Cæsar wrote a book for the third form ; what man who ever passed through the third form would now read Cæsar? Euclid wrote a book for the lower school ; a lower school book let it remain.

And $(a+b)^2 = a^2 + 2ab + b^2$ is equivalent to II. 4. And if $\frac{a}{b} = \frac{c}{d}$, then $\frac{ma \pm nb}{pa \pm qb} = \frac{mc \pm nd}{pc \pm qd}$, and this is equivalent to the immortal philosophy of the fifth book. "Great God, I'd rather be a pagan cradled in a creed outworn !" I would rather be utterly ignorant of all the wonderful literature and science of the last twenty-four centuries, even of the wonderful achievements of the last fifty years, than not to have the sense that our whole system of so-called education is as degrading to literature and philosophy as it is to English boys and men.

We are not the heirs of all the ages, and we shall not for very long remain in the foremost files of our time if we depend upon the schoolmasters. I place my faith in the common sense of the common people. In one way or another I find that they are learning to compute, to gain a knowledge of natural science. I know of many hundreds of night-school boys who were poor who are now successful engineers, and already youths are being

warned from trying to become engineers because their public school education would actually prevent their having a chance of success. They cannot understand the most elementary lectures in applied science. I know of a large employer who has already told the headmaster of a great public school that he will no longer employ public school boys unless a more rational method of teaching mathematics is adopted. And he is a public school boy himself ! I am constantly being asked to recommend men to teach mathematics in technical schools and colleges, and warned that I must not recommend a Cambridge man. There is nobody who has a higher respect for Cambridge mathematics, for the achievements of past and present Cambridge men, than I have ; but if Cambridge men will put themselves altogether out of sympathy with the needs of young engineers ; if they will make no attempt whatsoever to look at things from the new point of view to which we have been forced ; if without any attempt at examination they will in an off-hand way settle it that what we ask for is an illogical and soul-debasing non-educational preparation of an olla podrida of mere formulæ, then in sorrow and not without some anger we must try to get on without them. They do not know what a lovely bit of fighting they are leaving us to do all by ourselves, but I sincerely hope that they will not hamper us. Indeed, they must sooner or later help us against the common enemy, even if they are only to be armed as were the children of the mist. Because Isaac Newton was such a superb bowman and the English yew was ever the finest of materials, they will insist on the use of the antiquated weapon only. I sincerely hope that the English yew, which is very much of a graveyard tree, may not yet flourish over the grave of British industry.

But enough of these notions. I see a great fight ahead of our people, and bows and arrows are better than no weapons, as a twentieth of a loaf is better than no bread at all, and I welcome any instalment of reform, however small, in the teaching of mathematics in the public schools of England. And so long as my help is not rejected on the ground that I openly ask for a much greater reform and may be dangerous to my friends on that account, so long am I anxious to give my help and proud that it should be accepted. JOHN PERRY.

Birds attacking Butterflies and Moths.

WITH reference to my previous letter in NATURE (January 16), I would say that the butterfly referred to was the *Terias silhetana* or *Terias laeta*, probably both.

Another bird that frequently catches these butterflies on the wing is the Indian Bee Eater (*Merops viridis*).

During a Christmas camp this season I came across a field where some twenty or thirty King Crows were busily engaged in catching butterflies ; the day I first saw them, butterflies were numerous in this field, and it was easy to get undamaged specimens of *Terias silhetana*, *Terias laeta*, *Junonia lemonias*, *Tarucus theophrastus*, *Lampides elpis*, *Catopsilia pyranthe*, and some others which were not being caught in flight. Some three or four days later few King Crows were to be seen, the butterflies were much diminished in number, and nearly all those caught were damaged specimens. The birds perched on the tall dry Jowari stalks and made short flights on all sides, catching their prey sometimes on the wing, sometimes on the ground.

I could not say with certainty what butterflies were caught on the wing.

The King Crow and the Bee Eater are two of the commonest birds in this part of the country, and must cause a good deal of destruction in the course of a year. ANNIE E. McKAY.

India, February 21.

"Nature-Study" Exhibition.

WILL you kindly permit me, while thanking you for the attention which you have already directed towards the above exhibition, to state that it has now been arranged to hold it at